## Quantum dot devices for Cavity quantum electrodynamics

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## Quantum Dot (QD) as quantum light emitters



 $E_{XX} \neq E_X$  Coulomb interactions

Single photon emission, entanglement, quantum cryptography...

Problem: how to efficiently collect the photons?

## Single photon emitter in vacuum



Numerical aperture of an optical system (lens):

N.A. = 
$$n \sin(\theta_{\text{Max}}) = n \sin\left[\tan^{-1}\left(\frac{D}{2f}\right)\right]$$

Let's assume an isotropic radiation pattern:

$$\frac{\Omega}{4\pi} = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\theta_{\text{Max}}} \sin\theta d\theta = \frac{1}{2} (1 - \cos\theta_{\text{Max}})$$



Example: microscope objective (N.A = 0.55)  $\theta_{\rm Max} = 33.4^{\circ}$ 

Up to 8% collection efficiency.

## Extraction from a semiconductor material



 $\sin\theta_2 = n_1 \sin\theta_1$ 

Total internal reflection:

$$\theta \ge \theta_{\mathrm{Max}} = \sin^{-1}\left(\frac{1}{n_1}\right)$$

New J. Phys. 6, 96 (2004)

Example:  $\theta_{Max} = 16.6^{\circ}$ .  $\leq 2\%$  extraction efficiency (optimistic estimate).

Collection efficiency limited by our capacity to extract light from the material!

# Integrated optics in semiconductor materials



R. Trotta, Sapienza University (Italy)



P. Senellart, C2N (France)

## Solid immersion lens (SIL)



Add hemisphere  $(1 \le n_1 \le n_2)$  to modify total internal reflection condition. Light extraction possible for:

$$\theta \le \theta_{\mathrm{Max}} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Example: LaSFN9 glass ( $n_2 = 1.83$ )  $\theta_{Max} = 31^{\circ}$  and  $\approx 7.4$  % extraction efficiency.



Ultimately, carve the lens directly into GaAs (focused ion beam milling):  $n_1 \le n_2$ .

 $\theta_{Max} = 90^{\circ}$ .

50% of the light is still lost  $\Rightarrow$  add a mirror?

## Integrated mirrors

General ideal: Stack different materials generate wave interference



Notion of "photonic crystal".

## Reflection and transmission coefficients at an interface

Semiconductor materials with different indices of refraction  $\Rightarrow$  Fresnel reflection.

Normal incidence



Continuity of tangential components for  $\boldsymbol{E}$ :  $E_i + E_r = E_t$ 

Continuity of tangential components for **B**:

$$\frac{n_1}{c}E_i - \frac{n_1}{c}E_r = \frac{n_2}{c}E_t$$

Reflection coefficient:

$$=\frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

Transmission coefficient:  $t = \frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$ 

r

#### Homework: redo the calculation for any incidence angle.

## Distributed Bragg Reflector (DBR)

General ideal: Use Fresnel reflections  $(n_1 > n_2)$  to generate wave interference



Optical path  $\lambda_0/4$  for every layer  $\Rightarrow$  strong reflection coefficient (mirror).

## **Distributed Bragg Reflector (DBR)**



$$R(\lambda_0) \simeq 1 - 4\left(\frac{n_2}{n_1}\right)^{2N}$$

Example:  $n_1 = 3.5$  (GaAs)  $n_2 = 2.9$  (AlAs) N = 30 pairs  $\lambda_0 = 850$  nm

 $d_1 = 60.7 \text{ nm}$  $d_2 = 73.3 \text{ nm}$  $R(\lambda_0) \approx 0.99995$ 





pubs.acs.org/journal/apchd5

### Single Quantum Dot with Microlens and 3D-Printed Micro-objective as Integrated Bright Single-Photon Source

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# Light trapping in an optical microcavity

## Fabry Pérot resonator



## Quality factor

Full width half max:  $k_0^{\pm} = \pm \frac{1}{L_0} \frac{1}{2(R_1 R_2)^{1/4}} (1 - \sqrt{R_1 R_2})$   $\implies \delta \omega = \frac{1}{\pi} \frac{1 - R}{\sqrt{R}} \Delta \omega$ 

Quality factor:
$$Q = \frac{\omega_0}{\delta \omega} = \frac{2L_0}{\lambda_0} \frac{\pi \sqrt{R}}{1-R}$$
Photon lifetime: $\tau_{cav} = \frac{Q}{\omega_0}$ 

$$E(\vec{k}) = \hbar c \sqrt{\left(\frac{p\pi}{L_0}\right)^2 + k_{\parallel}^2}$$
$$\simeq E_0 + \frac{\hbar^2 k_{\parallel}^2}{2m_{\text{eff}}}$$

Parabolic dispersion relation.
 Effective mass for the photon.

$$\left|\vec{k}\right| = \frac{\omega}{c}$$



## Semiconductor microcavities



## Semiconductor "photonic dots"

![](_page_16_Picture_1.jpeg)

(1,0) (1,1) (1,2) (2,0)

![](_page_16_Picture_3.jpeg)

![](_page_16_Figure_4.jpeg)

![](_page_16_Figure_5.jpeg)

5 µm

Two-level system coupled to a single mode of the electromagnetic field

## The model

![](_page_18_Figure_1.jpeg)

![](_page_18_Figure_2.jpeg)

Two-level system

![](_page_18_Figure_4.jpeg)

Single mode of the electromagnetic field

![](_page_18_Figure_6.jpeg)

Microcavity

![](_page_18_Picture_8.jpeg)

## The model

![](_page_19_Figure_1.jpeg)

## Jaynes-Cummings model

QD basis:  $\{|g\rangle; |e\rangle\}$ QD basis:  $\{|g\rangle; |e\rangle\}$ Coupled system: tensor productPhoton number state basis:  $\{|n\rangle\}$  $\{|g\rangle; |e\rangle\} \otimes \{|n\rangle\}$ 

Hamiltonian:  $\hat{H} = \hbar \omega_0 |e\rangle \langle e| + \hbar \omega_0 \hat{a}^{\dagger} \hat{a} + \hat{H}_{int}$ 

Guess for the interaction Hamiltonian:

![](_page_20_Figure_5.jpeg)

## Interaction Hamiltonian

$$\hat{H}_{\text{int}} = -\frac{q}{m}\hat{\vec{p}}\cdot\hat{\vec{A}}$$
$$\hat{\vec{A}} = \sqrt{\frac{\hbar}{2\epsilon_0\omega V}}\vec{\epsilon}(\hat{a}^{\dagger} + \hat{a})$$
$$\hat{\vec{A}} = \sqrt{\frac{\hbar}{2\epsilon_0\omega V}}\vec{\epsilon}(\hat{a}^{\dagger} + \hat{a})$$

Momentum operator:

$$\langle g | \hat{\vec{p}} \cdot \vec{\epsilon} | g \rangle = \langle e | \hat{\vec{p}} \cdot \vec{\epsilon} | e \rangle = 0$$
 (integral of an odd function).  
 $\hat{\vec{p}} \cdot \vec{\epsilon} = \langle e | \hat{\vec{p}} \cdot \vec{\epsilon} | g \rangle [|e \rangle \langle g| + |g \rangle \langle e|]$ 

$$\implies \hat{H}_{\text{int}} = -\frac{q}{m} \sqrt{\frac{\hbar}{2\epsilon_0 \omega V}} \langle e | \, \hat{\vec{p}} \cdot \vec{\epsilon} \, |g\rangle \left( |e\rangle \, \langle g| + |g\rangle \, \langle e| \right) \otimes \left( \hat{a}^{\dagger} + \hat{a} \right)$$

 $\hat{H}_{\text{int}} = \hbar \Omega \left( \ket{e} \langle g \ket{\otimes \hat{a}} + \ket{g} \langle e \ket{\otimes \hat{a}^{\dagger}} \right)$  (resonant terms only).

## Matrix elements

## $\hat{a}^{\dagger} |n\rangle = \sqrt{n+1} |n+1\rangle$ $\hat{a} |n\rangle = \sqrt{n} |n-1\rangle$

 $\langle g, n | \hat{H}_{\text{int}} | g, n' \rangle = 0$ 

$$\langle e, n | \hat{H}_{\text{int}} | e, n' \rangle = 0$$

$$\begin{aligned} \langle g, n | \hat{H}_{\text{int}} | e, n' \rangle &= \hbar \Omega \left( \langle g | e \rangle \langle g | e \rangle \langle n | \hat{a} | n' \rangle + \langle g | g \rangle \langle e | e \rangle \left\langle n | \hat{a}^{\dagger} | n' \right\rangle \right) \\ &= \hbar \Omega \sqrt{n' + 1} \langle n | n' + 1 \rangle \\ &= \hbar \Omega \sqrt{n} \delta_{n,n'+1} \\ \langle e, n' | \hat{H}_{\text{int}} | g, n \rangle &= \hbar \Omega \sqrt{n} \delta_{n,n'+1} \end{aligned}$$

![](_page_23_Figure_0.jpeg)

## Diagonalization of the Hamiltonian

Diagonalization of the 2x2 Hamiltonian:  $\hat{H}_n = \begin{pmatrix} n\hbar\omega_0 & \hbar\Omega\sqrt{n} \\ \hbar\Omega\sqrt{n} & n\hbar\omega_0 \end{pmatrix}$ 

Eigenstates and eigenvalues:

$$\begin{cases} \lambda_{+} = n\hbar\omega_{0} + \hbar\Omega\sqrt{n} \\ \lambda_{-} = n\hbar\omega_{0} - \hbar\Omega\sqrt{n} \end{cases} \begin{cases} |+,n\rangle = (|g,n\rangle + |e,n-1\rangle)/\sqrt{2} \\ |-,n\rangle = (|g,n\rangle - |e,n-1\rangle)/\sqrt{2} \end{cases}$$

![](_page_24_Figure_4.jpeg)

#### Article

## A gated quantum dot strongly coupled to an optical microcavity

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![](_page_25_Figure_7.jpeg)

![](_page_25_Figure_8.jpeg)

![](_page_26_Figure_0.jpeg)

## Photon blockade

Interaction with the two-level system ⇒ Anharmonic energy ladder

![](_page_26_Figure_3.jpeg)

Laser resonant with  $|-,1\rangle \Rightarrow$  quantum statistics (photon antibunching).

## Dynamics of the coupled system

How does the excited quantum dot decays into an empty cavity?

Initial state: 
$$|i\rangle = |e, 0\rangle = \frac{1}{\sqrt{2}} (|+, 1\rangle - |-, 1\rangle)$$
  
After time t:  $|f\rangle = \frac{1}{\sqrt{2}} \left( |+, 1\rangle e^{-i(\hbar\omega_0 + \hbar\Omega)t/\hbar} - |-, 1\rangle e^{-i(\hbar\omega_0 - \hbar\Omega)t/\hbar} \right)$   
 $= \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} (|g, 1\rangle + |e, 0\rangle) e^{-i\Omega t} - \frac{1}{\sqrt{2}} ((|g, 1\rangle - |e, 0\rangle)) e^{i\Omega t} \right) e^{-i\omega_0 t}$   
 $= (-i |g, 1\rangle \sin \Omega t + |e, 0\rangle \cos \Omega t) e^{-i\omega_0 t}$ 

 $\implies P_e(t) = |\cos^2 \Omega t|$ 

No decay, but periodic exchange of energy between atom and cavity: "Rabi oscillation".

![](_page_27_Figure_5.jpeg)

## A gated quantum dot strongly coupled to an optical microcavity

![](_page_28_Figure_3.jpeg)

Initial state:

$$|i\rangle = |2, -\rangle = \frac{1}{\sqrt{2}} \left(|g, 2\rangle - |e, 1\rangle\right)$$

Fig. 3 | Time-resolved vacuum Rabi oscillations. Intensity autocorrelation

Destruction of one photon at t=0+: 
$$\frac{\hat{a}|i\rangle}{||\hat{a}|i\rangle||} = \frac{1}{\sqrt{3}} \left(\frac{\sqrt{2}-1}{\sqrt{2}}|+,1\rangle + \frac{\sqrt{2}+1}{\sqrt{2}}|+,1\rangle\right)$$

At time t: 
$$|f\rangle = \frac{1}{\sqrt{3}}e^{-i\omega_0 t} \left[ \left(\sqrt{2}\cos\Omega t + i\sin\Omega t\right) |g,1\rangle - \left(\cos\Omega t + i\sqrt{2}\sin\Omega t\right) |e,0\rangle \right]$$

Probability of photon Destruction at time t:  $P(t) = |\hat{a}|f\rangle|^2 = \frac{1}{3}(1 + \cos^2 \Omega t)$ 

## Generalization to the case of a detuned cavity

![](_page_29_Figure_1.jpeg)

Exercise: Calculate the corresponding eigenvalues and eigenvectors.

![](_page_29_Figure_3.jpeg)

## Intermediate conclusions

- Quantum dot coupling a single mode of the electromagnetic field leads to:
  - New (entangled) states for the coupled system,
  - Anharmonic energy spectrum (antibunching),
  - Modified emission (Rabi oscillations, ...).

![](_page_30_Picture_5.jpeg)

- Ideal model. Not a realistic situation:
  - What about losses? Coupling to the environment?
  - How does the system decay?

## Effect of the cavity losses, Purcell effect

### Master equation

![](_page_32_Figure_1.jpeg)

Master equation for the density matrix, including damping:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} \left[ \hat{H}_1, \hat{\rho} \right] - \frac{\kappa}{2} (\hat{a}^{\dagger} \hat{a} \hat{\rho} + \hat{\rho} \hat{a}^{\dagger} \hat{a}) + \kappa \hat{a} \hat{\rho} \hat{a}^{\dagger}$$
$$\kappa = \tau_c^{-1} = \omega_0 / Q$$

Exercise: show that:

$$\frac{d}{dt} \begin{bmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{12} - \rho_{21} \end{bmatrix} = \begin{bmatrix} 0 & 0 & i\Omega \\ 0 & -\kappa & -i\Omega \\ 2i\Omega & -2i\Omega & -\kappa/2 \end{bmatrix} \begin{bmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{12} - \rho_{21} \end{bmatrix}$$

## Master equation

![](_page_33_Figure_1.jpeg)

## Cavity decay rate

$$\Gamma_{\rm cav} = |\Lambda_+| \simeq 4\Omega^2 \frac{Q}{\omega_0}$$

With  $\Omega$  defined in:  $\hat{H}_{\text{int}} = -\frac{q}{m} \sqrt{\frac{\hbar}{2\epsilon_0 \omega V}} \langle e | \, \hat{\vec{p}} \cdot \vec{\epsilon} \, | g \rangle \left( | e \rangle \, \langle g | + | g \rangle \, \langle e | \right) \otimes \left( \hat{a}^{\dagger} + \hat{a} \right)$ 

One obtains: 
$$\Gamma_{cav} = 2 \frac{q^2}{m^2} \frac{\left| \langle e | \, \hat{\vec{p}} \cdot \vec{\epsilon} \, | g \rangle \right|^2}{\epsilon_0 \hbar} \frac{Q}{V \omega_0^2}$$
  
To be compared to:  $\Gamma_{vac} = \frac{1}{3\pi} \frac{q^2}{m^2} \frac{\left| \langle e | \, \hat{\vec{p}} \cdot \vec{\epsilon} \, | g \rangle \right|^2}{\epsilon_0 \hbar} \frac{\omega_0}{c^3}$ 

 $\Rightarrow$  Modification of the emission rate by the cavity: **PURCELL EFFECT**.

## Fermi golden rule

![](_page_35_Figure_1.jpeg)

Excited state coupled to a continuum of radiation modes:

Fermi golden rule.

Transition probability:  $\Gamma = \frac{2\pi}{\hbar^2} \left| \langle g, 1_k | \hat{H}_{int} | e, 0 \rangle \right|^2 \rho(\omega)$ 

![](_page_35_Figure_5.jpeg)

#### Emission rate and directionality enhanced at resonance.

**B10.** Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University.*—For nuclear magnetic moment transitions at radio frequencies the probability of spontaneous emission, computed from

$$A_{\nu} = (8\pi\nu^2/c^3)h\nu(8\pi^3\mu^2/3h^2)$$
 sec.<sup>-1</sup>,

is so small that this process is not effective in bringing a spin system into thermal equilibrium with its surroundings. At 300°K, for  $\nu = 10^7$  sec.<sup>-1</sup>,  $\mu = 1$  nuclear magneton, the corresponding relaxation time would be  $5 \times 10^{21}$  seconds! However, for a system coupled to a resonant electrical circuit, the factor  $8\pi\nu^2/c^3$  no longer gives correctly the number of radiation oscillators per unit volume, in unit frequency range, there being now one oscillator in the frequency range  $\nu/Q$  associated with the circuit. The spontaneous emission probability is thereby increased, and the relaxation time reduced, by a factor  $f = 3Q\lambda^3/4\pi^2 V$ , where V is the volume of the resonator. If a is a dimension characteristic of the circuit so that  $V \sim a^3$ , and if  $\delta$  is the skin-depth at frequency  $\nu$ ,  $f \sim \lambda^3/a^2 \delta$ . For a non-resonant circuit  $f \sim \lambda^3/a^3$ , and for  $a < \delta$  it can be shown that  $f \sim \lambda^3/a\delta^2$ . If small metallic particles, of diameter  $10^{-3}$  cm are mixed with a nuclear-magnetic medium at room temperature, spontaneous emission should establish thermal equilibrium in a time of the order of minutes, for  $\nu = 10^7$  sec.<sup>-1</sup>.

## First demonstration with InAs quantum dots

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PHYSICAL REVIEW LETTERS

3 AUGUST 1998

#### **Enhanced Spontaneous Emission by Quantum Boxes in a Monolithic Optical Microcavity**

J. M. Gérard,\* B. Sermage, B. Gayral, B. Legrand, E. Costard,<sup>†</sup> and V. Thierry-Mieg<sup>‡</sup>

France Télécom/CNET/DTD/CDP, 196 avenue H. Ravera, F-92220 Bagneux, France (Received 3 April 1998)

![](_page_37_Figure_7.jpeg)

### nature nanotechnology

LETTERS

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#### A bright and fast source of coherent single photons

Natasha Tomm<sup>®</sup><sup>1,3</sup>, Alisa Javadi<sup>®</sup><sup>1,3</sup><sup>∞</sup>, Nadia Olympia Antoniadis<sup>®</sup><sup>1</sup>, Daniel Najer<sup>®</sup><sup>1</sup>, Matthias Christian Löbl<sup>®</sup><sup>1</sup>, Alexander Rolf Korsch<sup>®</sup><sup>1,2</sup>, Rüdiger Schott<sup>2</sup>, Sascha René Valentin<sup>2</sup>, Andreas Dirk Wieck<sup>®</sup><sup>2</sup>, Arne Ludwig<sup>®</sup><sup>2</sup> and Richard John Warburton<sup>®</sup><sup>1</sup>

A single-photon source is an enabling technology in device-independent quantum communication<sup>1</sup>, quantum simulation<sup>2,3</sup>, and linear optics-based<sup>4</sup> and measurement-based quantum computing<sup>5</sup>. These applications employ many photons and place stringent requirements on the efficiency of single-photon creation. The scaling on efficiency is typically an exponential function of the number of photons. Schemes taking full advantage of quantum superpositions also depend sensitively on the coherence of the photons, that is, their indistinguishability<sup>6</sup>. Here, we report a single-photon source with a high end-to-end efficiency. We employ gated quantum dots in an open, tunable microcavity<sup>7</sup>. The gating provides control of the charge and electrical tuning of the emission frequency; the high-quality material ensures low noise; and the tunability of the microcavity compensates for the lack of control in quantum dot position and emission frequency. Transmission through the top mirror is the dominant escape route for photons from the microcavity, and this output is well matched to a single-mode fibre. With this design, we can create a single photon at the output of the final optical fibre on-demand with a probability of up to 57% and with an average two-photon interference visibility of 97.5%. Coherence persists in trains of thousands of photons with single-photon creation at a repetition rate of 1 GHz.

![](_page_38_Figure_6.jpeg)

![](_page_38_Figure_7.jpeg)

## Micropillar sources for quantum technologies

![](_page_39_Figure_1.jpeg)

Micropillar sources developed in P. Senellart's group (C2N):

• **2014:** extraction efficiencies ≈ 53 %

![](_page_39_Picture_4.jpeg)

![](_page_39_Picture_5.jpeg)

![](_page_39_Picture_6.jpeg)

## The problem of scalability

Random assembling, positioning of quantum dots during fabrication:

Inhomogeneous broadening

![](_page_40_Figure_3.jpeg)

#### Solution for 1 single photon source:

In-situ optical lithography

![](_page_40_Figure_6.jpeg)

PRL 101, 267404 (2008)

#### No existing technologies for arrays of identical single photon sources!

## Conclusion

- Using **integrated optics** to solve the photon extraction issue.
- Most efficient way to extract photons: quantum dot in cavity.
- Jaynes Cummings model: description of lightmatter coupling in an idealistic scenario.
- Coupling to the environment needs to be added to describe emission properties.
- Nearly optimal single photon sources available today.
- Scalability issue: work on the technology? Use 2D active materials?