# Exciton-polaritons: resonant drive and interactions (II)

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### Driven-dissipative Gross-Pitaevskii equation

Full Hamiltonian for the lower polaritons (k-space):

$$\hat{H}_{\rm LP} \simeq \sum_{\mathbf{k}} \hbar \omega_X(\mathbf{k}) \hat{p}_{\mathbf{k}}^{\dagger} \hat{p}_{\mathbf{k}} + \frac{V_{\mathbf{0}}^{XX}}{2} |X_0|^4 \sum_{\mathbf{k}, \mathbf{k'q}} \hat{p}_{k+q}^{\dagger} \hat{p}_{k-q}^{\dagger} \hat{p}_k \hat{p}_k$$

Full Hamiltonian for the lower polaritons (real-space):

$$\hat{H}_{\rm LP} = -\frac{\hbar^2}{2m_{\rm LP}} \int d^2 \mathbf{r} \hat{\Psi}_{\rm LP}^{\dagger}(\mathbf{r}) \nabla_{\mathbf{r}}^2 \hat{\Psi}_{\rm LP}(\mathbf{r}) + |X_0|^4 \frac{V_0^{XX}}{2} \int d^2 \mathbf{r} \hat{\Psi}_{\rm LP}^{\dagger}(\mathbf{r}) \hat{\Psi}_{\rm LP}^{\dagger}(\mathbf{r}) \hat{\Psi}_{\rm LP}(\mathbf{r}) \hat{\Psi}_{\rm LP}(\mathbf{r})$$

Heisenberg equation for the field operator:

$$i\hbar\frac{d}{dt}\hat{\Psi}_{\rm LP}(\mathbf{r},t) = -\frac{\hbar^2}{2m_{\rm LP}}\nabla_{\mathbf{r}}^2\hat{\Psi}_{\rm LP}(\mathbf{r},t) + U\hat{\Psi}_{\rm LP}^{\dagger}(\mathbf{r},t)\hat{\Psi}_{\rm LP}(\mathbf{r},t)\hat{\Psi}_{\rm LP}(\mathbf{r},t)$$

Mean field approximation (classical field):

$$i\hbar\frac{d}{dt}\Psi_{\rm LP}(\mathbf{r},t) = -\frac{\hbar^2}{2m_{\rm LP}}\nabla_{\mathbf{r}}^2\Psi_{\rm LP}(\mathbf{r},t) + U\Psi_{\rm LP}^*(\mathbf{r},t)\Psi_{\rm LP}(\mathbf{r},t)\Psi_{\rm LP}(\mathbf{r},t)$$

### Driven-dissipative Gross-Pitaevskii equation

Add terms for dissipation and laser drive:

"Driven-dissipative Gross Pitaevskii equation":

$$i\frac{d}{dt}\psi = \left[\omega_0 - \frac{\hbar}{2m}\nabla^2 + gn - i\frac{\gamma}{2}\right]\psi + iF$$

## **Multi-stability**

Scanning the laser energy at fixed laser power.



 $\omega_p - \omega_{LP}$ 

### **Multi-stability**



### Stability of the solutions

Driven-dissipative GPE: 
$$i\frac{d}{dt}\psi = \left[\omega_0 - \frac{\hbar}{2m}\nabla^2 + gn - i\frac{\gamma}{2}\right]\psi + iF$$

Assume **k=0**,  $F = \sqrt{\frac{\gamma}{2}} F_0 e^{-i\omega_{\rm p}t}$  and search solutions of the form  $\psi(\mathbf{r}, t) = \psi_{\rm ss} e^{-i\omega_{\rm p}t}$ 

Steady-state solution: 
$$\left[\omega_{\rm p} - (\omega_0 + g |\psi_{\rm ss}|^2) + i\frac{\gamma}{2}\right]\psi_{\rm ss} = i\sqrt{\frac{\gamma}{2}}F_0$$
$$\left[(\omega_p - (\omega_0 + gN_{\rm LP}))^2 + \left(\frac{\gamma}{2}\right)^2\right]N_{\rm LP} = \frac{\gamma}{2}|F_0|^2$$

Consider small perturbation on top of steady-state solution:

$$\psi(\mathbf{r},t) = \sqrt{N_{\rm LP}}e^{-i\omega_{\rm p}t} + \delta\psi_{\rm LP}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_{\rm p}t)}$$

Steady-state solution of GPE

### Linearized GPE

$$i\partial_t \delta \psi = \left(\omega_0 + \frac{\hbar k^2}{2m_{\rm LP}} - \omega_{\rm p} - i\frac{\gamma}{2}\right)\delta\psi + \underline{2gN_{\rm LP}\delta\psi + gN_{\rm LP}\delta\psi^*}$$

Only keep first order terms in  $\delta\psi$  and  $\delta\psi^*$ 

### Linearized GPE

$$\begin{split} \dot{i}\partial_t \delta\psi &= \left(\omega_0 + \frac{\hbar k^2}{2m_{\rm LP}} - \omega_{\rm p} - i\frac{\gamma}{2}\right)\delta\psi + 2gN_{\rm LP}\delta\psi + gN_{\rm LP}\delta\psi^* \\ -i\partial_t \delta\psi^* &= \left(\omega_0 + \frac{\hbar k^2}{2m_{\rm LP}} - \omega_{\rm p} + i\frac{\gamma}{2}\right)\delta\psi^* + 2gN_{\rm LP}\delta\psi^* + gN_{\rm LP}\delta\psi \\ \dot{i}\partial_t \left(\frac{\delta\psi}{\delta\psi^*}\right) &= \left(\begin{bmatrix}\omega_{\rm LP} + \frac{\hbar k^2}{2m_{\rm LP}} + 2gN_{\rm LP}\end{bmatrix} - \omega_{\rm p} - i\frac{\gamma}{2} & gN_{\rm LP} \\ -gN_{\rm LP} & \omega_{\rm p} - \left[\omega_{\rm LP} + \frac{\hbar k^2}{2m_{\rm LP}} + 2gN_{\rm LP}\right] - i\frac{\gamma}{2}\right)\left(\frac{\delta\psi}{\delta\psi^*}\right) \\ &i\partial_t \left(\frac{\delta\psi}{\delta\psi^*}\right) = \mathcal{L}_{\rm Bog}\left(\frac{\delta\psi}{\delta\psi^*}\right) \end{split}$$

Linearized equation.

Unstable solution when at least one of the eigenvalues has a positive imaginary part.

### Linearized GPE

$$i\partial_t \delta \psi = \left(\omega_0 + \frac{\hbar k^2}{2m_{\rm LP}} - \omega_{\rm p} - i\frac{\gamma}{2}\right) \delta \psi + 2gN_{\rm LP}\delta\psi + gN_{\rm LP}\delta\psi^*$$
$$-i\partial_t \delta \psi^* = \left(\omega_0 + \frac{\hbar k^2}{2m_{\rm LP}} - \omega_{\rm p} + i\frac{\gamma}{2}\right) \delta \psi^* + 2gN_{\rm LP}\delta\psi^* + gN_{\rm LP}\delta\psi$$

$$i\partial_t \begin{pmatrix} \delta\psi\\ \delta\psi^* \end{pmatrix} = \begin{pmatrix} \left[\omega_{\rm LP} + \frac{\hbar k^2}{2m_{\rm LP}} + 2gN_{\rm LP}\right] - \omega_{\rm p} - i\frac{\gamma}{2} & gN_{\rm LP}\\ -gN_{\rm LP} & \omega_{\rm p} - \left[\omega_{\rm LP} + \frac{\hbar k^2}{2m_{\rm LP}} + 2gN_{\rm LP}\right] - i\frac{\gamma}{2} \end{pmatrix} \begin{pmatrix} \delta\psi\\ \delta\psi^* \end{pmatrix}$$
$$i\partial_t \begin{pmatrix} \delta\psi\\ \delta\psi^* \end{pmatrix} = \mathcal{L}_{\rm Bog} \begin{pmatrix} \delta\psi\\ \delta\psi^* \end{pmatrix}$$

$$M = \begin{pmatrix} \omega_{LP} + 2 * g * n - \omega_p - I * \gamma/2 & g * n \\ -g * n & -\omega_{LP} - 2 * g * n + \omega_p - I * \gamma/2 \end{pmatrix};$$

(\*MatrixForm[M]\*)

FullSimplify[Eigenvalues[M]]

$$\left\{-\frac{\mathbb{i}\gamma}{2} - \sqrt{(g n + \omega_{LP} - \omega_{p}) (3 g n + \omega_{LP} - \omega_{p})} , -\frac{\mathbb{i}\gamma}{2} + \sqrt{(g n + \omega_{LP} - \omega_{p}) (3 g n + \omega_{LP} - \omega_{p})} \right\}$$

### Unstable solutions

$$\omega_{\text{Bog}} = -i\frac{\gamma}{2} \pm \sqrt{(\omega_{\text{LP}}(k) + gN_{LP} - \omega_{\text{P}})(\omega_{\text{LP}}(k) + 3gN_{LP} - \omega_{\text{P}})}$$

Stability condition at *k*=0:

$$(gN_{\rm LP} - \Delta)(3gN_{\rm LP} - \Delta) \le -\frac{\gamma^2}{4}$$
 (with  $\Delta = \omega_{\rm p} - \omega_{\rm LP}$ )  
 $3(gN_{\rm LP})^2 - 4\Delta(gN_{\rm LP}) + \Delta^2 + \frac{\gamma^2}{4} \le 0$ 

Second order polynomial in  $gN_{LP}$  with real roots for positive discriminant:

$$16\Delta^2 - 12(\Delta^2 + \frac{\gamma^2}{4}) \ge 0 \quad \Rightarrow \quad \Delta \ge \frac{\sqrt{3}}{2} \quad \text{or} \quad \Delta \le -\frac{\sqrt{3}}{2}$$
  
Two real roots: 
$$gN_{\text{LP}}^{\pm} = \frac{2}{3}\Delta \pm \frac{1}{3}\sqrt{\Delta^2 - 3\left(\frac{\gamma}{2}\right)^2}$$

Positive only when  $\Delta \ge 0$ 

### **Unstable solutions**

Existence of unstable solutions when:

$$\omega_{\rm p} \ge \omega_{\rm LP} + \frac{\sqrt{3}}{2}\gamma$$

Obtained in the range:

$$\frac{2}{3}\Delta - \frac{1}{3}\sqrt{\Delta^2 - 3\left(\frac{\gamma}{2}\right)^2} \le gN_{\rm LP} \le \frac{2}{3}\Delta + \frac{1}{3}\sqrt{\Delta^2 - 3\left(\frac{\gamma}{2}\right)^2}$$

### **Unstable solutions**

One unstable solution  $\Rightarrow$  **BISTABILITY** 



LOW EXCITATION POWER ( $N_{LP} = 0.1$ )



## Bogoliubov spectrum of excitations LOW EXCITATION POWER ( $N_{LP} = 0.1$ )

$$gN_{\rm LP} \ll \Delta \Rightarrow \omega_{\rm Bog} \simeq -i\frac{\gamma}{2} \pm [\omega_{\rm LP}(k) - \omega_{\rm P}] \Rightarrow \Re[\omega_{\rm Bog}] \simeq \omega_{\rm LP}(k) - \omega_{\rm P}$$

 $\Rightarrow$  One recovers the polariton dispersion (linear regime).

LOW EXCITATION POWER ( $N_{LP} = 0.1$ )

$$gN_{\rm LP} \ll \Delta \Rightarrow \omega_{\rm Bog} \simeq -i\frac{\gamma}{2} \pm [\omega_{\rm LP}(k) - \omega_{\rm P}] \Rightarrow \Re[\omega_{\rm Bog}] \simeq \omega_{\rm LP}(k) - \omega_{\rm P}$$



HIGH EXCITATION POWER ( $N_{LP} = 27$ )



### HIGH EXCITATION POWER ( $N_{LP} = 27$ )

 $gN_{\rm LP} \gg \Delta \Rightarrow$  Taylor expansion for small k

$$\frac{\hbar k^2}{2m_{\rm LP}} - \Delta \ll g N_{\rm LP}$$

$$\omega_{\text{Bog}} = -i\frac{\gamma}{2} \pm \left(\frac{\hbar}{2m_{\text{LP}}}k^2 + gN_{LP} - \Delta\right)^{1/2} \left(\frac{\hbar}{2m_{\text{LP}}}k^2 + 3gN_{LP} - \Delta\right)^{1/2}$$
$$\simeq -i\frac{\gamma}{2} \pm \sqrt{3}gN_{\text{LP}} \left[ \left(1 + \frac{1}{2gN_{\text{LP}}} \left(\frac{\hbar k^2}{2m_{\text{LP}}} - \Delta\right)\right) \left(1 + \frac{1}{6gN_{\text{LP}}} \left(\frac{\hbar k^2}{2m_{\text{LP}}} - \Delta\right)\right) \right]$$
$$\simeq -i\frac{\gamma}{2} \pm \left[\frac{2}{\sqrt{3}}(\omega_{\text{LP}}(k) - \omega_{\text{p}}) + \sqrt{3}gN_{\text{LP}}\right]$$
$$\Re[\omega_{\text{Bog}}] \simeq \pm \left[\frac{2}{\sqrt{3}}(\omega_{\text{LP}}(k) - \omega_{\text{p}}) + \sqrt{3}gN_{\text{LP}}\right]$$

 $\Rightarrow$  Two parabolic dispersion shifted by the interaction.

HIGH EXCITATION POWER ( $N_{LP} = 27$ )

$$gN_{\rm LP} \gg \Delta \Rightarrow \Re[\omega_{\rm Bog}] \simeq \pm \left[\frac{2}{\sqrt{3}}(\omega_{\rm LP}(k) - \omega_{\rm p}) + \sqrt{3}gN_{\rm LP}\right]$$



POLARITON POPULATION WITHIN UNSTABLE RANGE ( $N_{LP} = 13$ )



POLARITON POPULATION WITHIN UNSTABLE RANGE ( $N_{LP} = 13$ )

 $\Rightarrow$  Expect positive imaginary part next to k = 0.



**MEDIUM EXCITATION POWER**  $(gN_{LP} \approx \Delta)$ 



## Bogoliubov spectrum of excitations **MEDIUM EXCITATION POWER** ( $gN_{LP} \approx \Delta$ )

$$\omega_{\rm Bog} = -i\frac{\gamma}{2} \pm \sqrt{(\omega_{\rm LP}(k) + gN_{LP} - \omega_{\rm P})(\omega_{\rm LP}(k) + 3gN_{LP} - \omega_{\rm P})}$$

$$gN_{\rm LP} \simeq \Delta \quad \Rightarrow \quad \omega_{\rm Bog} \simeq -i\frac{\gamma}{2} \pm \sqrt{\omega_{\rm LP}(k)(\omega_{\rm LP}(k) + 2gN_{\rm LP})}$$

## Taylor expansion (large k) $\omega_{ m LP}(k) \gg 2g N_{ m LP}$

 $\Rightarrow \Re[\omega_{\rm Bog}] \simeq \pm [\omega_{\rm LP}(k) + gN_{\rm LP}]$ 

Parabolic dispersion

### Taylor expansion (small k)

 $2gN_{\rm LP}\gg\omega_{\rm LP}(k)$ 

$$\Rightarrow \Re[\omega_{\rm Bog}] \simeq \pm \sqrt{\frac{\hbar}{m}} g N_{\rm LP} \times k$$

Linear dispersion

 $\Rightarrow$  Phonon-like dispersion

"Speed of sound":  $c_{
m s}=\sqrt{\hbar g N_{
m LP}/m_{
m LP}}$ 

**MEDIUM EXCITATION POWER**  $(gN_{LP} \approx \Delta)$ 

$$gN_{\rm LP} \simeq \Delta \quad \Rightarrow \quad \omega_{\rm Bog} \simeq -i\frac{\gamma}{2} \pm \sqrt{\omega_{\rm LP}(k)(\omega_{\rm LP}(k) + 2gN_{\rm LP})}$$



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### OPEN

# Dispersion relation of the collective excitations in a resonantly driven polariton fluid

Petr Stepanov<sup>1</sup>, Ivan Amelio<sup>2</sup>, Jean-Guy Rousset<sup>1,3</sup>, Jacqueline Bloch<sup>4</sup>, Aristide Lemaître<sup>1</sup>, Alberto Amo<sup>5</sup>, Anna Minguzzi<sup>6</sup>, Iacopo Carusotto<sup>2</sup> & Maxime Richard<sup>1</sup>



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### The special case of sonic dispersion relation



No available Bogoliubov mode at the pump energy ⇒ Consequences for polariton superfluidity

## Polariton superfluidity

Superfluidity is "the characteristic property of a fluid with zero viscosity which therefore flows without any loss of kinetic energy"



**Induce polariton flow**  $\Rightarrow$  pump at  $k_p \neq 0$ 

Modified Bogoliubov dispersion:

$$\omega_{\rm Bog} = (\mathbf{k} - \mathbf{k}_{\rm p}) \cdot \frac{\hbar \mathbf{k}_{\rm p}}{m_{\rm LP}} - i\frac{\gamma}{2} \pm \sqrt{\left(\omega_{\rm LP}(\mathbf{k}_{\rm p}) + \frac{\hbar (\mathbf{k} - \mathbf{k}_{\rm p})^2}{2m_{\rm LP}} + gN_{\rm LP} - \omega_{\rm p}\right) \left(\omega_{\rm LP}(\mathbf{k}_{\rm p}) + \frac{\hbar (\mathbf{k} - \mathbf{k}_{\rm p})^2}{2m_{\rm LP}} + 3gN_{\rm LP} - \omega_{\rm p}\right)}$$

Subsonic flow  $V_{\rm p} < C_{\rm s}$ 

$$gN_{\rm LP} \simeq \Delta \quad \Rightarrow \quad \omega_{\rm Bog}(\mathbf{k}) \simeq (\mathbf{k} - \mathbf{k}_{\rm p}) \cdot \frac{\hbar \mathbf{k}_{\rm p}}{m_{\rm LP}} \pm c_{\rm s} \left| \mathbf{k} - \mathbf{k}_{\rm p} \right|$$



## Subsonic flow $V_{\rm p} < C_{\rm s}$

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$$gN_{\rm LP} \simeq \Delta \quad \Rightarrow \quad \omega_{\rm Bog}(\mathbf{k}) \simeq (\mathbf{k} - \mathbf{k}_{\rm p}) \cdot \frac{\hbar \mathbf{k}_{\rm p}}{m_{\rm LP}} \pm c_{\rm s} \left| \mathbf{k} - \mathbf{k}_{\rm p} \right|$$





#### **Probing Microcavity Polariton Superfluidity through Resonant Rayleigh Scattering**

Iacopo Carusotto<sup>1,2,\*</sup> and Cristiano Ciuti<sup>3</sup>

Subsonic case <sup>1</sup>Laboratoire Kastler Brossel, École Normale Supérieure, 24 rue Lhomond, 75005 Parts <sup>2</sup>CRS BEC-INFM and Dipartimento di Fisica, Università di Trento, I-38050 Povo, Italy <sup>3</sup>Laboratoire Pierre Aigrain, École Normale Supérieure, 24 rue Lhomond, 75005 Paris, France (Received 23 April 2004; published 13 October 2004)

We study the motion of a polariton fluid injected into a planar microcavity by a continuous wave laser. In the presence of static defects, the spectrum of the Bogoliubov-like excitations reflects onto the shape and intensity of the resonant Rayleigh scattering emission pattern in both momentum and real space.



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### Bogoliubov-Čerenkov Radiation in a Bose-Einstein Condensate Flowing against an Obstacle

I. Carusotto,<sup>1</sup> S. X. Hu,<sup>2,\*</sup> L. A. Collins,<sup>2</sup> and A. Smerzi<sup>2,1</sup>

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Supersonic case We study the density modulation that appears in a Bose-Einstein condensate flowing with supersonic velocity against an obstacle.



Conical density modulation downstream of the defect: Cerenkov regime.

### Bogoliubov-Čerenkov Radiation in a Bose-Einstein Condensate Flowing against an Obstacle

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## Conclusion

- Cavity exciton-polaritons  $\Rightarrow$  "Fluids of light"
- Optical platform for driven-dissipative nonlinear hydrodynamics
- Superfluidity is just one example. Plenty of other interesting effects:
  - Solitons, vortices, parametric oscillations, acoustic black holes
- Study synthetic polaritonic matter in lattices (see next class)
- Prospects for quantum polaritonic (current research topic)