Exciton-polaritons: resonant drive and interactions

Sylvain Ravets

sylvain.ravets@c2n.upsaclay.fr

Master QLMN

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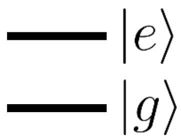
Reminder: 0D versus 2D

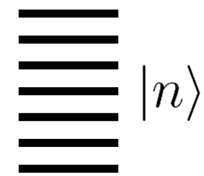
QD basis: $\{|g\rangle\,;|e\rangle\}$



$$\hat{H} = \hbar\omega_0 |e\rangle \langle e|$$

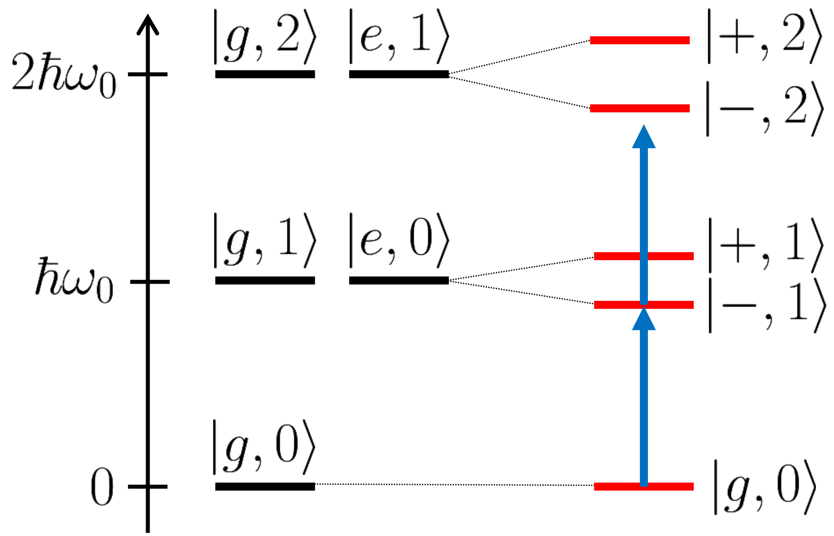
Two-level system

Photon number state basis: $\{|n\rangle\}$



$$\hat{H} = \hbar\omega_0 \hat{a}^\dagger \hat{a}$$

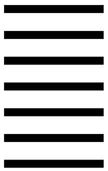
Bosonic operator (harmonic oscillator)



Anharmonic energy ladder

Reminder: 0D versus 2D

QW exciton basis: $\{|n_X\rangle\}$

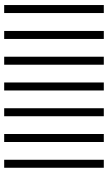


$|n_X\rangle$

$\hat{H} = \hbar\omega_X b^\dagger b$

Bosonic operator (harmonic oscillator)

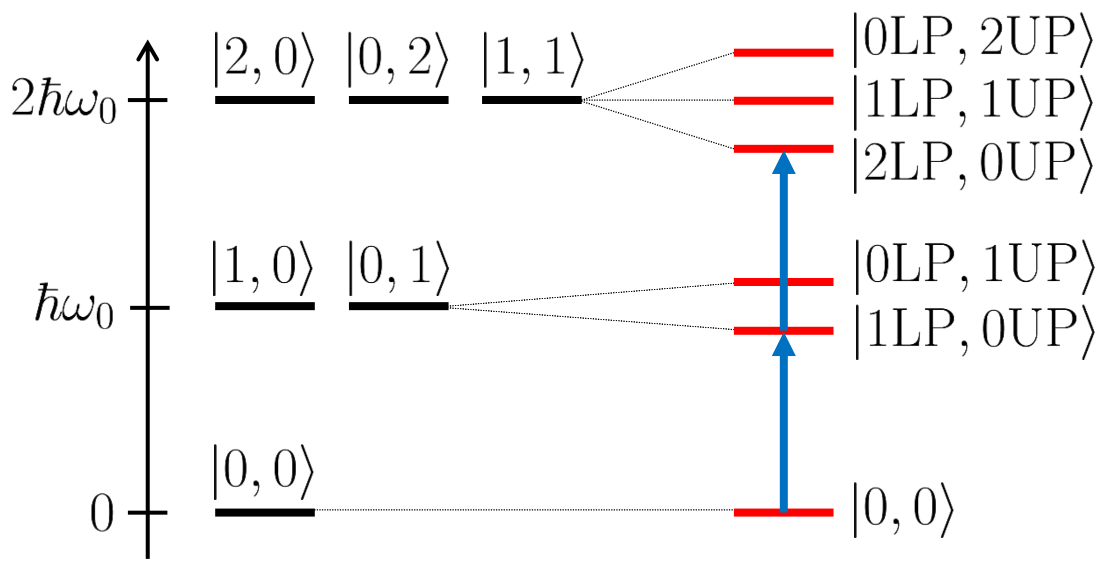
Photon number state basis: $\{|n\rangle\}$



$|n\rangle$

$\hat{H} = \hbar\omega_0 \hat{a}^\dagger \hat{a}$

Bosonic operator (harmonic oscillator)

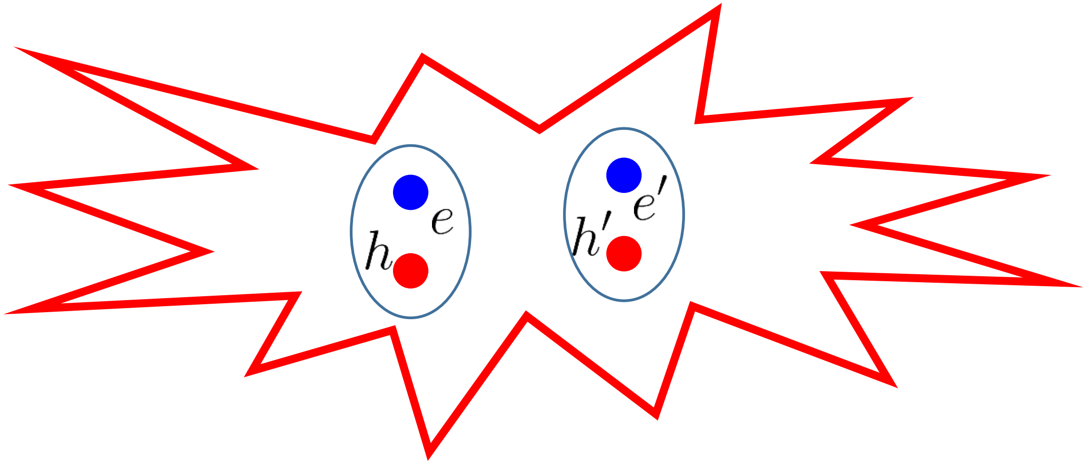


Harmonic energy ladder

Exciton-exciton interaction

PHYSICAL REVIEW B VOLUME 58, NUMBER 12 15 SEPTEMBER 1998-II

Role of the exchange of carriers in elastic exciton-exciton scattering in quantum wells



$$V(r)=e^2/(\epsilon_0 r)$$

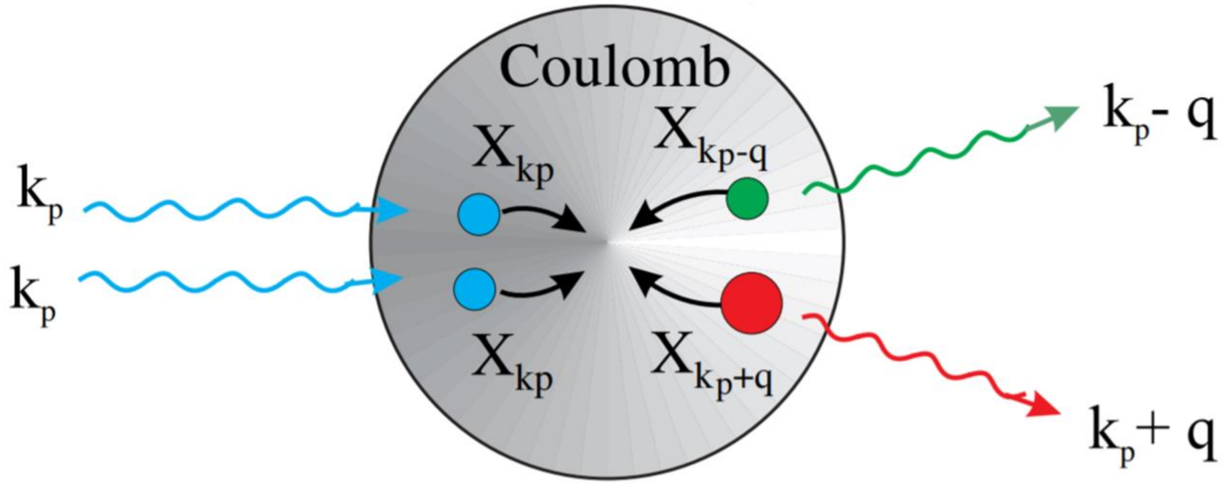
$$H=-\frac{\hbar^2}{2m_e}\nabla_e^2-\frac{\hbar^2}{2m_h}\nabla_h^2-\frac{\hbar^2}{2m_e}\nabla_{e'}^2$$

$$-\frac{\hbar^2}{2m_h}\nabla_{h'}^2-V(|\boldsymbol{r}_e-\boldsymbol{r}_h|)-V(|\boldsymbol{r}_{e'}-\boldsymbol{r}_{h'}|)+V(|\boldsymbol{r}_e-\boldsymbol{r}_{e'}|)$$

$$+V(|\boldsymbol{r}_h-\boldsymbol{r}_{h'}|)-V(|\boldsymbol{r}_e-\boldsymbol{r}_{h'}|)-V(|\boldsymbol{r}_h-\boldsymbol{r}_{e'}|)$$

Exciton-exciton interaction

Approximation: use a contact two-body interaction for the exciton.



Full Hamiltonian for the exciton:

$$\hat{H}_X = \sum_{\mathbf{k}} \hbar\omega_X(\mathbf{k})\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'\mathbf{q}} V^{XX}_{\mathbf{q}}\hat{b}^\dagger_{\mathbf{k}+\mathbf{q}}\hat{b}^\dagger_{\mathbf{k}'-\mathbf{q}}\hat{b}_{\mathbf{k}}\hat{b}_{\mathbf{k}'}$$

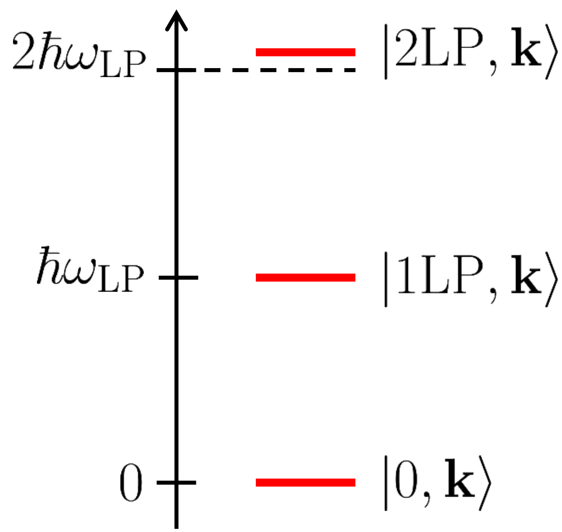
$$\simeq \sum_{\mathbf{k}} \hbar\omega_X(\mathbf{k})\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}} + \frac{V^{XX}_{\mathbf{0}}}{2}\sum_{\mathbf{k},\mathbf{k}'\mathbf{q}} \hat{b}^\dagger_{\mathbf{k}+\mathbf{q}}\hat{b}^\dagger_{\mathbf{k}'-\mathbf{q}}\hat{b}_{\mathbf{k}}\hat{b}_{\mathbf{k}'}$$

Lower polariton total Hamiltonian

Full Hamiltonian for the lower polaritons:

$$\hat{H}_{\mathrm{LP}} \simeq \sum_{\mathbf{k}} \hbar\omega_X(\mathbf{k})\hat{p}^\dagger_{\mathbf{k}}\hat{p}_{\mathbf{k}} + \frac{V_{\mathbf{0}}^{XX}}{2} \sum_{\mathbf{k},\mathbf{k}'\mathbf{q}} X^*_{k+q} X^*_{k-q} X_k X_k \hat{p}^\dagger_{k+q}\hat{p}^\dagger_{k-q}\hat{p}_k\hat{p}_k$$

$$\simeq \sum_{\mathbf{k}} \hbar\omega_X(\mathbf{k})\hat{p}^\dagger_{\mathbf{k}}\hat{p}_{\mathbf{k}} + \frac{V_{\mathbf{0}}^{XX}}{2} |X_0|^4 \sum_{\mathbf{k},\mathbf{k}'\mathbf{q}} \hat{p}^\dagger_{k+q}\hat{p}^\dagger_{k-q}\hat{p}_k\hat{p}_k$$



Energy blue-shift (repulsive).

Anharmonic energy ladder

Real space Hamiltonian

Real space Hamiltonian ⇒ Fourier transform: $\hat{p}_{\mathbf{k}} = \frac{1}{\sqrt{2\pi}^2} \int d^2\mathbf{r} \Psi_{\mathrm{LP}}(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$

$$\hat{H}_{\mathrm{LP}} = \underbrace{-\frac{\hbar^2}{2m_{\mathrm{LP}}} \int d^2\mathbf{r} \hat{\Psi}^{\dagger}_{\mathrm{LP}}(\mathbf{r}) \nabla^2_{\mathbf{r}} \hat{\Psi}_{\mathrm{LP}}(\mathbf{r})}_{\text{Kinetic energy}} + \underbrace{|X_0|^4 \frac{V_0^{XX}}{2} \int d^2\mathbf{r} \hat{\Psi}^{\dagger}_{\mathrm{LP}}(\mathbf{r}) \hat{\Psi}^{\dagger}_{\mathrm{LP}}(\mathbf{r}) \hat{\Psi}_{\mathrm{LP}}(\mathbf{r}) \hat{\Psi}_{\mathrm{LP}}(\mathbf{r})}_{\text{Interaction energy}}$$

Evolution equation

Goal: write an evolution equation for observable $\hat{\Psi}_{\mathrm{LP}}(\mathbf{r})$

Heisenberg equation: $i\hbar\frac{d}{dt}\hat{\Psi}_{\mathrm{LP}}(\mathbf{r}) = \left[\hat{\Psi}_{\mathrm{LP}}(\mathbf{r}), \hat{H}\right]$

Reminder: $\left[\hat{A}, \hat{B}\hat{C}\right] = \hat{B}\left[\hat{A}, \hat{C}\right] + \left[\hat{A}, \hat{B}\right]\hat{C}$

$$\left[\hat{\Psi}_{\mathrm{LP}}(\mathbf{r_1}), \hat{\Psi}_{\mathrm{LP}}(\mathbf{r_2})\right] = \delta(\mathbf{r_1} - \mathbf{r_2})$$

$$i\hbar\frac{d}{dt}\hat{\Psi}_{\mathrm{LP}}(\mathbf{r}, t) = -\frac{\hbar^2}{2m_{\mathrm{LP}}}\nabla^2_{\mathbf{r}}\hat{\Psi}_{\mathrm{LP}}(\mathbf{r}, t) + U\hat{\Psi}^{\dagger}_{\mathrm{LP}}(\mathbf{r}, t)\hat{\Psi}_{\mathrm{LP}}(\mathbf{r}, t)\hat{\Psi}_{\mathrm{LP}}(\mathbf{r}, t)$$

Mean field approximation

$$i\hbar\frac{d}{dt}\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t) = -\frac{\hbar^2}{2m_{\mathrm{LP}}}\nabla^2_{\mathbf{r}}\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t) + U\hat{\Psi}^\dagger_{\mathrm{LP}}(\mathbf{r},t)\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)$$

Assume macroscopic occupation in $|\psi\rangle$ such that $\hat{\mathcal{O}}(\mathbf{r},t) \simeq \langle\psi|\,\hat{\mathcal{O}}(\mathbf{r},t)\,|\psi\rangle$

⇒ Neglect quantum fluctuations and replace field operators by classical fields.

$$\begin{aligned} i\hbar\frac{d}{dt}\left\langle\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\right\rangle &= -\frac{\hbar^2}{2m_{\mathrm{LP}}}\nabla^2_{\mathbf{r}}\left\langle\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\right\rangle + U\left\langle\hat{\Psi}^\dagger_{\mathrm{LP}}(\mathbf{r},t)\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\right\rangle \\ &\simeq -\frac{\hbar^2}{2m_{\mathrm{LP}}}\nabla^2_{\mathbf{r}}\left\langle\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\right\rangle + U\left\langle\hat{\Psi}^\dagger_{\mathrm{LP}}(\mathbf{r},t)\right\rangle\left\langle\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\right\rangle\left\langle\hat{\Psi}_{\mathrm{LP}}(\mathbf{r},t)\right\rangle \end{aligned}$$

Mean field approximation

$$\boxed{i\hbar\frac{d}{dt}\Psi_{\mathrm{LP}}(\mathbf{r},t) = -\frac{\hbar^2}{2m_{\mathrm{LP}}}\nabla^2_{\mathbf{r}}\Psi_{\mathrm{LP}}(\mathbf{r},t) + U\Psi^*_{\mathrm{LP}}(\mathbf{r},t)\Psi_{\mathrm{LP}}(\mathbf{r},t)\Psi_{\mathrm{LP}}(\mathbf{r},t)}$$

$|\Psi_{\mathrm{LP}}(\mathbf{r},t)|^2 = n_{\mathrm{LP}}(\mathbf{r},t)$ (Polariton density)

Generalization to drive and dissipation

Add terms for dissipation (phenomenological) and laser drive:

$$i\frac{d}{dt}\psi(\mathbf{r},t) = \left[\omega_0 - \frac{\hbar}{2m}\nabla^2 + \frac{U}{\hbar}n_{\mathrm{LP}}(\mathbf{r},t) - i\frac{\gamma}{2}\right]\psi(\mathbf{r},t) + iF_{\mathrm{exc}}(\mathbf{r},t)$$

Excitation field

Nonlinear term (interactions)
"Kerr-type" non-linearity:
$\mathbf{P}_{\mathrm{NL}} \propto \chi^{(3)}\,|\mathbf{E}|^2\,\mathbf{E}$

Cavity losses

"Driven-dissipative Gross Pitaevskii equation":

$$\boxed{i\frac{d}{dt}\psi = \left[\omega_0 - \frac{\hbar}{2m}\nabla^2 + gn - i\frac{\gamma}{2}\right]\psi + iF}$$

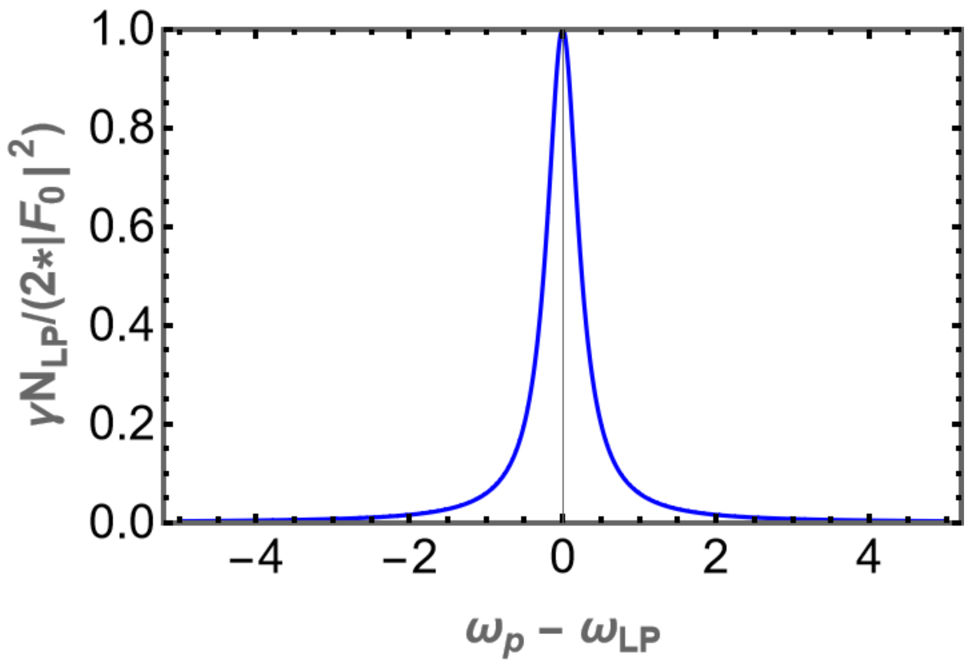
Solution in the linear regime (g=0)

Assume $F = \sqrt{\frac{\gamma}{2}} F_0 e^{i(\mathbf{k_p}\cdot\mathbf{r}-\omega_\mathrm{p} t)}$ and search solutions of the form: $\psi = \psi_\mathrm{ss} e^{i(\mathbf{k_p}\cdot\mathbf{r}-\omega_\mathrm{p} t)}$

One obtains: $\omega_\mathrm{p}\psi_\mathrm{ss} = \left[\omega_0(\mathbf{k_p}) - i\frac{\gamma}{2}\right]\psi_\mathrm{ss} + i\sqrt{\frac{\gamma}{2}}F_0$

$$\Rightarrow \quad |\psi_\mathrm{ss}|^2 = \frac{\sqrt{\frac{\gamma}{2}}\,|F_0|^2}{(\omega_\mathrm{p} - \omega_0(\mathbf{k_p}))^2 + \left(\frac{\gamma}{2}\right)^2}$$

(lorentzian profile)



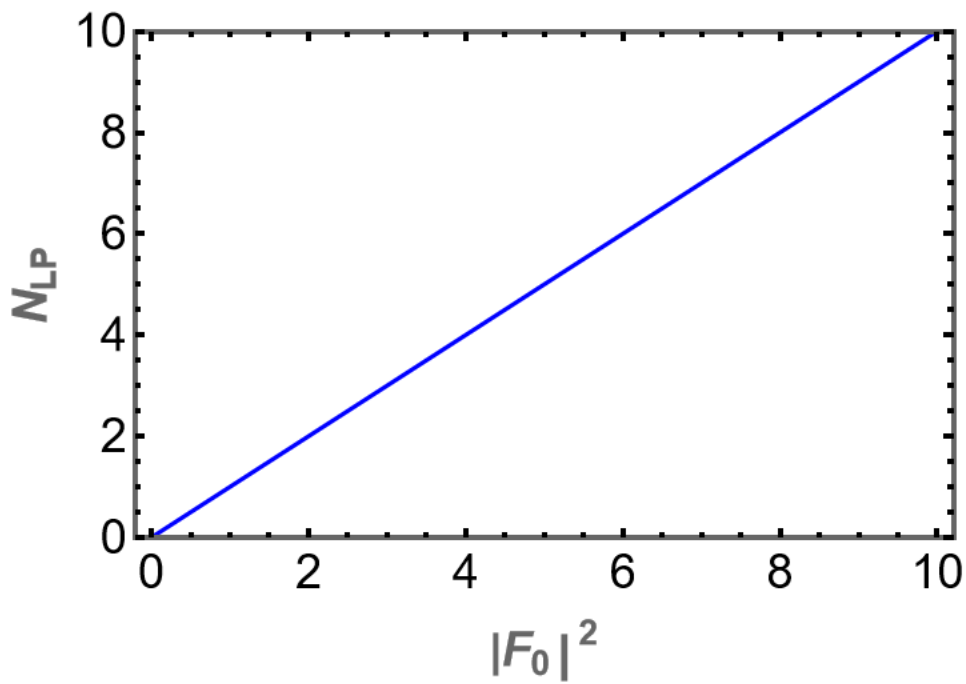
Varying the pump frequency around ω_LP at fixed pump amplitude (**$\mathbf{k_p}$=0**).

Solution in the linear regime (g=0)

Assume $F = \sqrt{\frac{\gamma}{2}} F_0 e^{i(\mathbf{k_p}\cdot\mathbf{r} - \omega_\mathrm{p} t)}$ and search solutions of the form: $\psi = \psi_\mathrm{ss} e^{i(\mathbf{k_p}\cdot\mathbf{r} - \omega_\mathrm{p} t)}$

One obtains: $\omega_\mathrm{p}\psi_\mathrm{ss} = \left[\omega_0(\mathbf{k_p}) - i\frac{\gamma}{2}\right]\psi_\mathrm{ss} + i\sqrt{\frac{\gamma}{2}} F_0$

$$\Rightarrow \quad |\psi_\mathrm{ss}|^2 = \frac{\sqrt{\frac{\gamma}{2}}\,|F_0|^2}{(\omega_\mathrm{p} - \omega_0(\mathbf{k_p}))^2 + \left(\frac{\gamma}{2}\right)^2}$$ (lorentzian profile)



Varying the pump amplitude for fixed pump frequency ($\omega_\mathrm{p} = \omega_\mathrm{LP}$).

Solution in nonlinear regime (g=0.1γ)

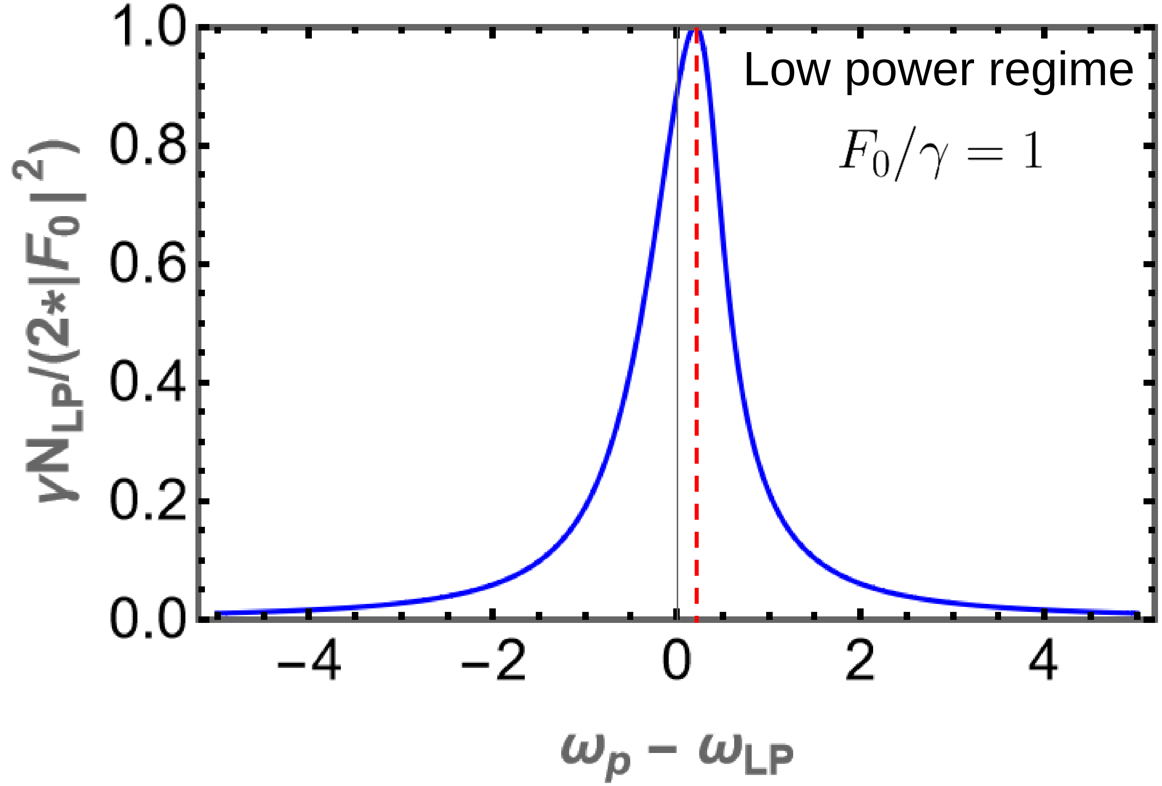
$$\begin{cases} \left[\omega_{\mathrm{p}} - (\omega_0 + g\,|\psi_{\mathrm{ss}}|^2) + i\frac{\gamma}{2}\right]\psi_{\mathrm{ss}} = i\frac{\gamma}{2}F_0 \\ \left[\omega_{\mathrm{p}} - (\omega_0 + g\,|\psi_{\mathrm{ss}}|^2) - i\frac{\gamma}{2}\right]\psi_{\mathrm{ss}}^* = -i\frac{\gamma}{2}F_0^* \end{cases}$$

$$\Rightarrow \boxed{\left[(\omega_p - \underbrace{(\omega_0 + gN_{\mathrm{LP}})}_{\text{Lower polariton energy “renormalized” by the interaction}})^2 + \left(\frac{\gamma}{2}\right)^2\right] N_{\mathrm{LP}} = \frac{\gamma}{2}\,|F_0|^2}$$

Third order polynomial equation for N_{LP} (nonlinear).

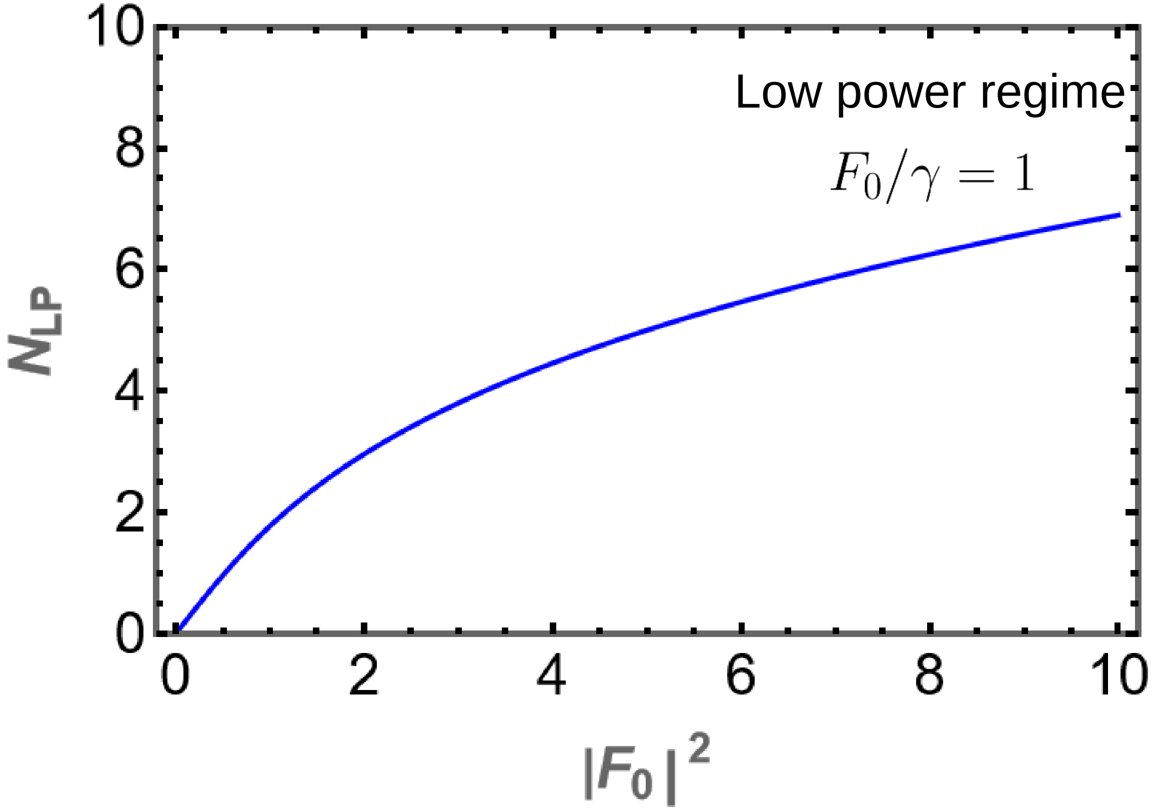
Solution in nonlinear regime (g=0.1γ)

```
N_LP2[ω_] := Solve[N*((ω - (0 + 0.1*N))^2 + (1/2)^2) - 1/2*1 == 0, N];
Plot[{N /. N_LP2[ω][[1]], N /. N_LP2[ω][[2]], N /. N_LP2[ω][[3]]}, {ω, -5, 5}, PlotRange → {0, 2},
 Frame → {True, True, True, True}, FrameStyle → Thick,
 FrameLabel → {Style["ωp - ωLP", 18, Bold], Style["NLP/|F0|^2", 18, Bold]},
 FrameTicksStyle → Directive[Black, 18]]
```



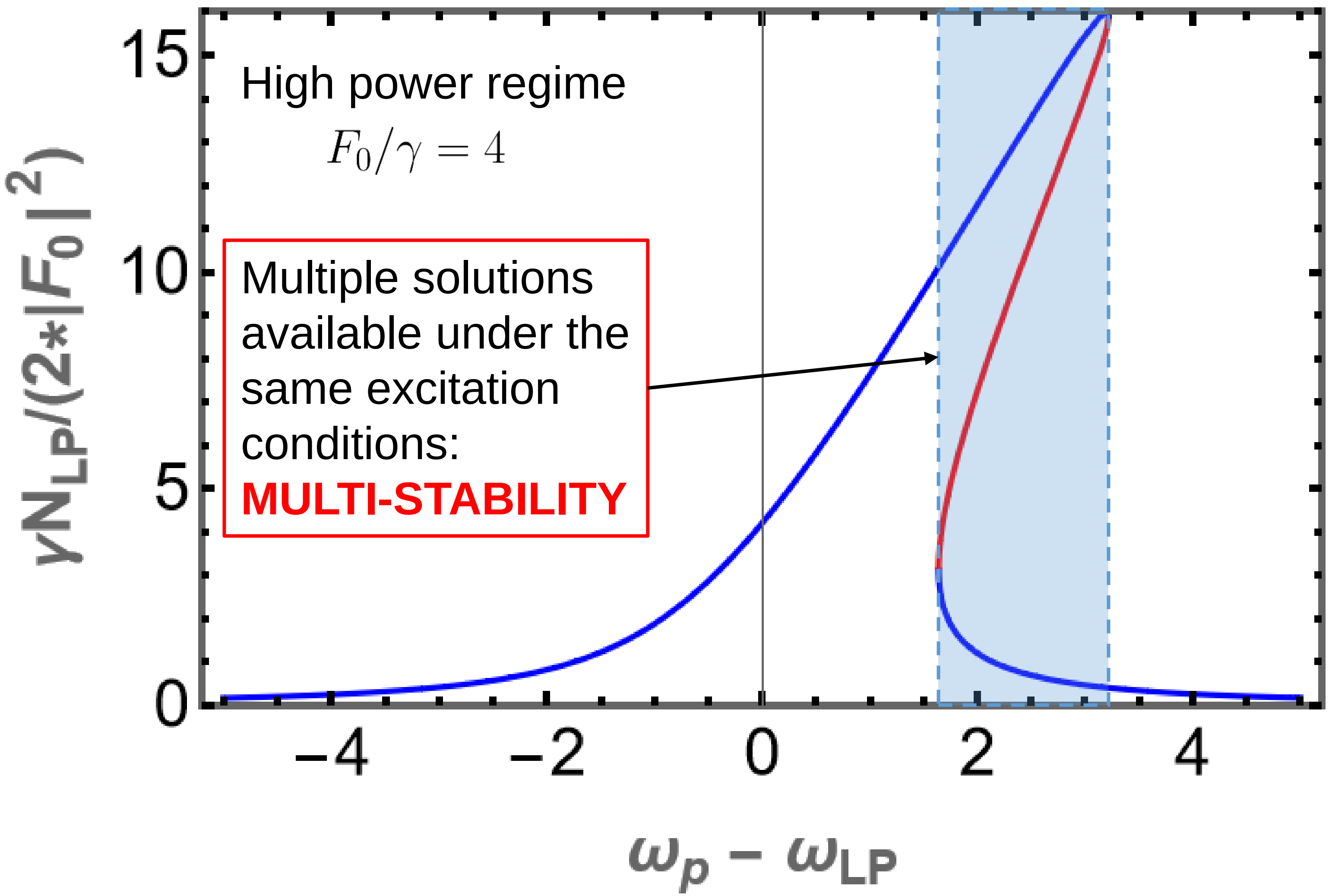
Solution in nonlinear regime (g=0.1γ)

```
N_LP2[I0_] := Solve[N * ((0 - (0 + 0.1 * N))^2 + (1/2)^2) - 1/2 * I0 == 0, N];
Plot[{N /. N_LP2[ω][[1]], N /. N_LP2[ω][[2]], N /. N_LP2[ω][[3]]}, {ω, 0, 10}, PlotRange → {0, 10},
 Frame → {True, True, True, True}, FrameStyle → Thick, PlotStyle → {Blue, Red, Blue},
 FrameLabel → {Style["|F_0|^2", 18, Bold], Style["N_LP", 18, Bold]},
 FrameTicksStyle → Directive[Black, 18]]
```



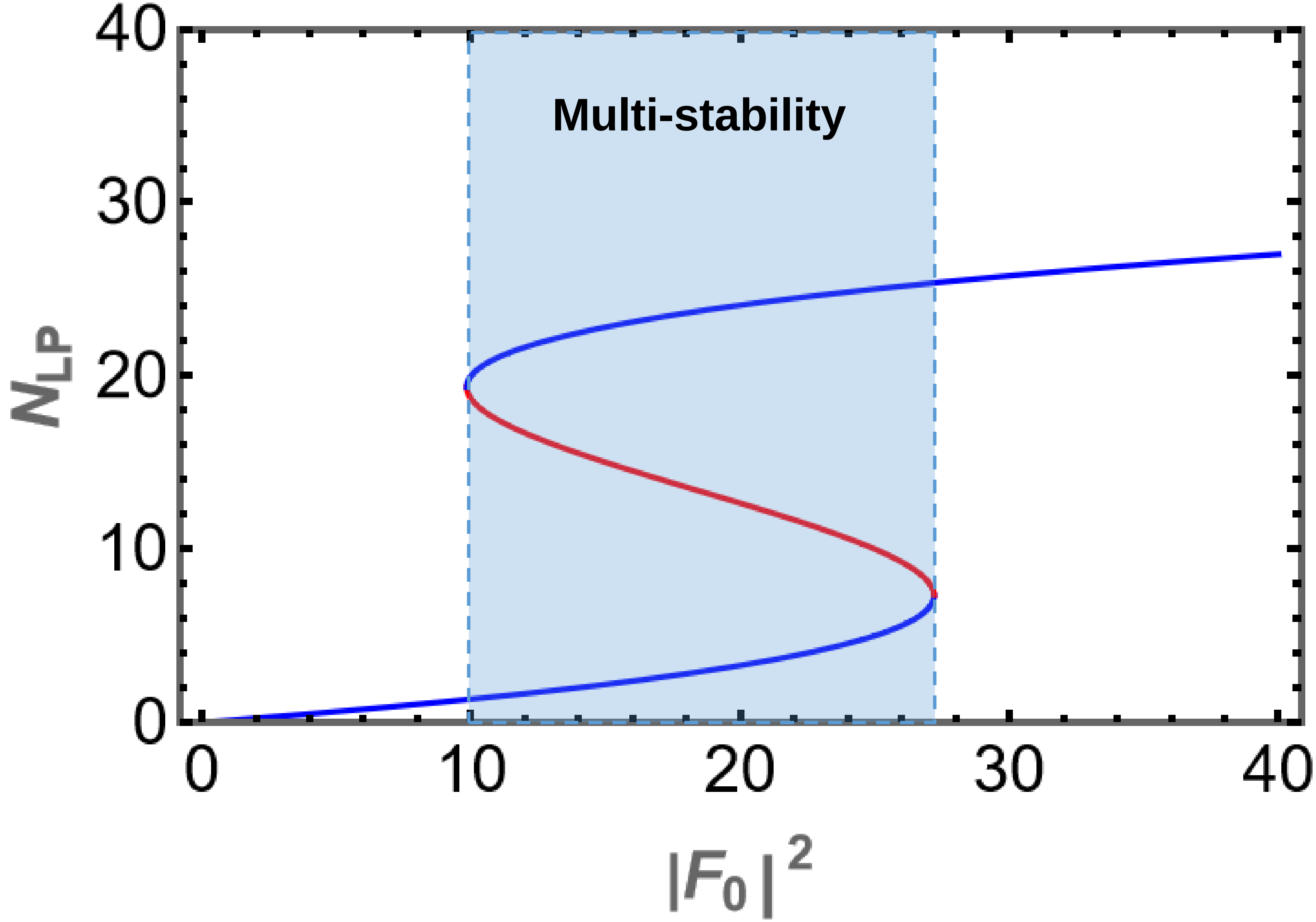
Multi-stability at higher power

Scanning the laser energy at fixed laser power.



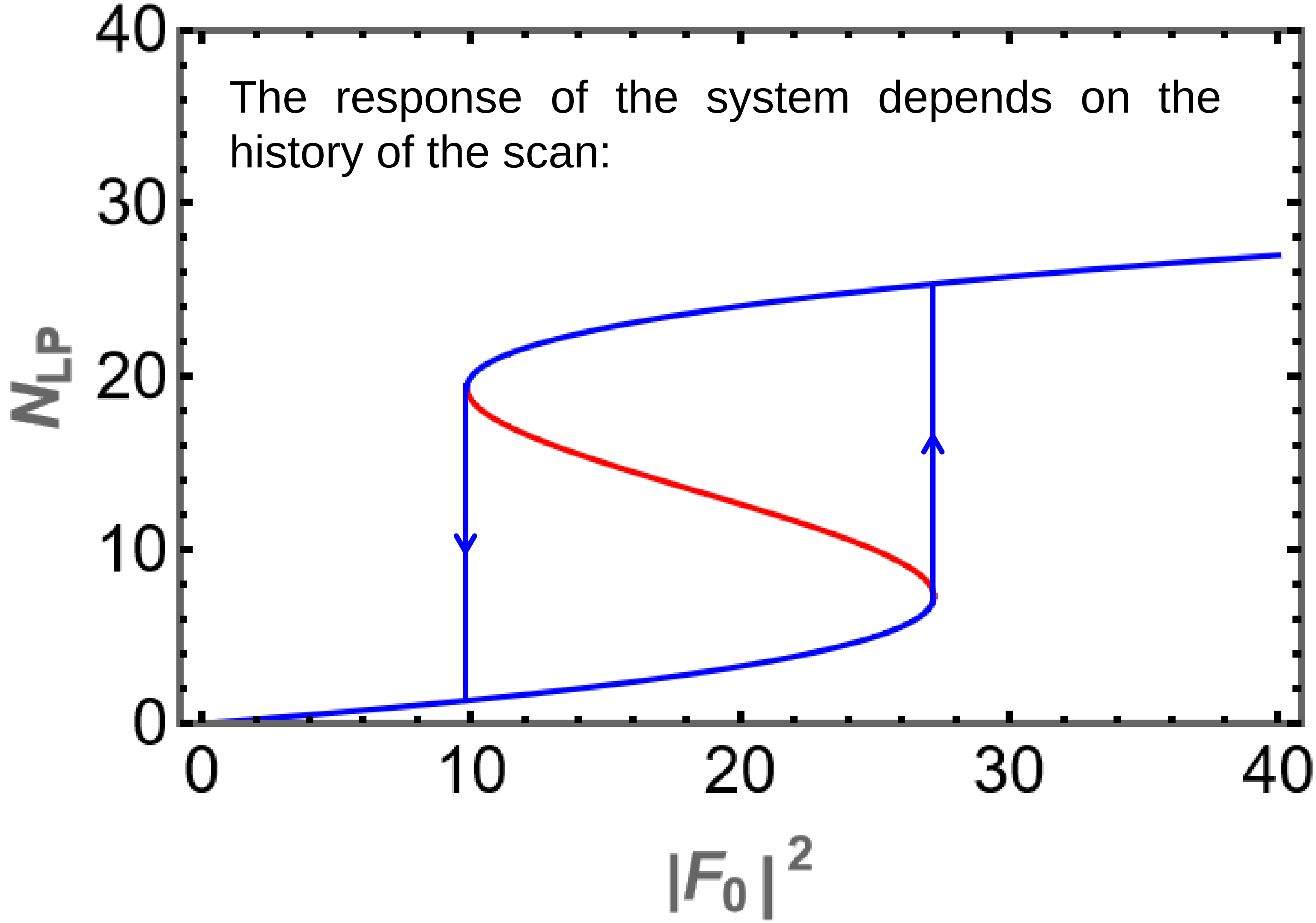
Multi-stability at higher power

Scanning the laser power at fixed laser energy ($\hbar\omega_p = \hbar\omega_{LP} + 2\gamma$).



Multi-stability at higher power

Scanning the laser power at fixed laser energy ($\hbar\omega_p=\hbar\omega_{LP}+2\gamma$).



First experimental demonstration

PHYSICAL REVIEW B **70**, 161307(R) (2004)

Optical bistability in semiconductor microcavities in the nondegenerate parametric oscillation regime: Analogy with the optical parametric oscillator

A. Baas, J.-Ph. Karr, M. Romanelli, A. Bramati, and E. Giacobino
Laboratoire Kastler Brossel, Université Paris 6, Ecole Normale Supérieure et CNRS, UPMC Case 74, 4 Place Jussieu, 75252 Paris Cedex 05, France

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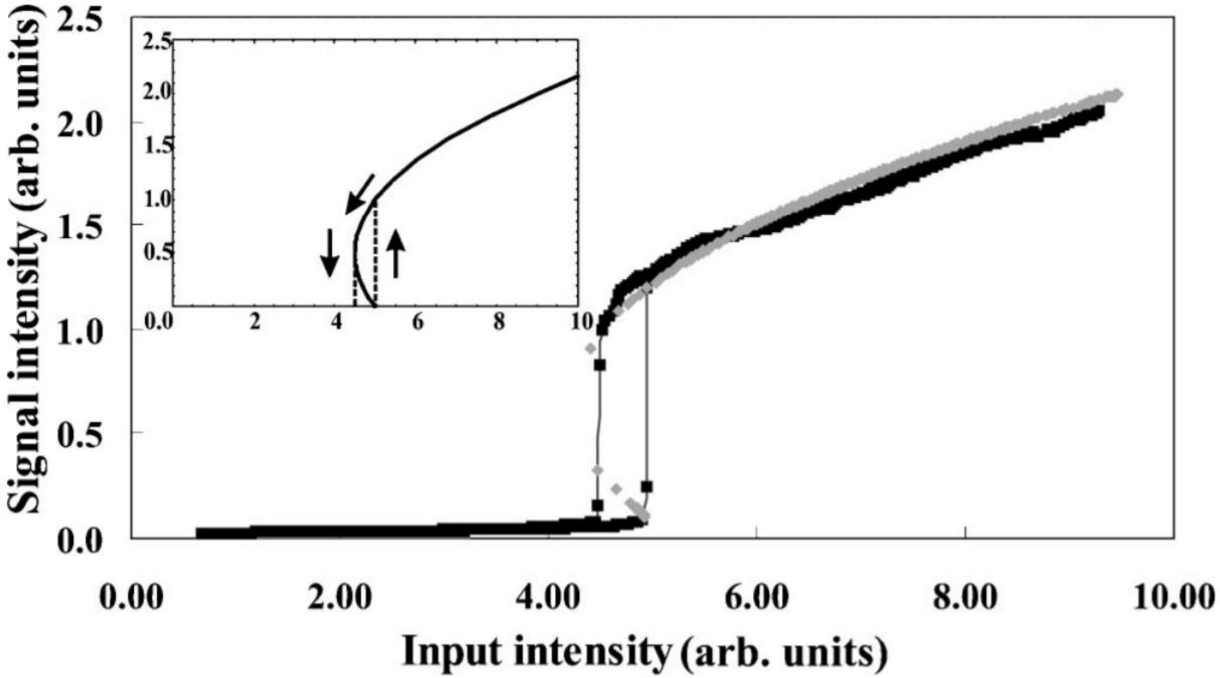
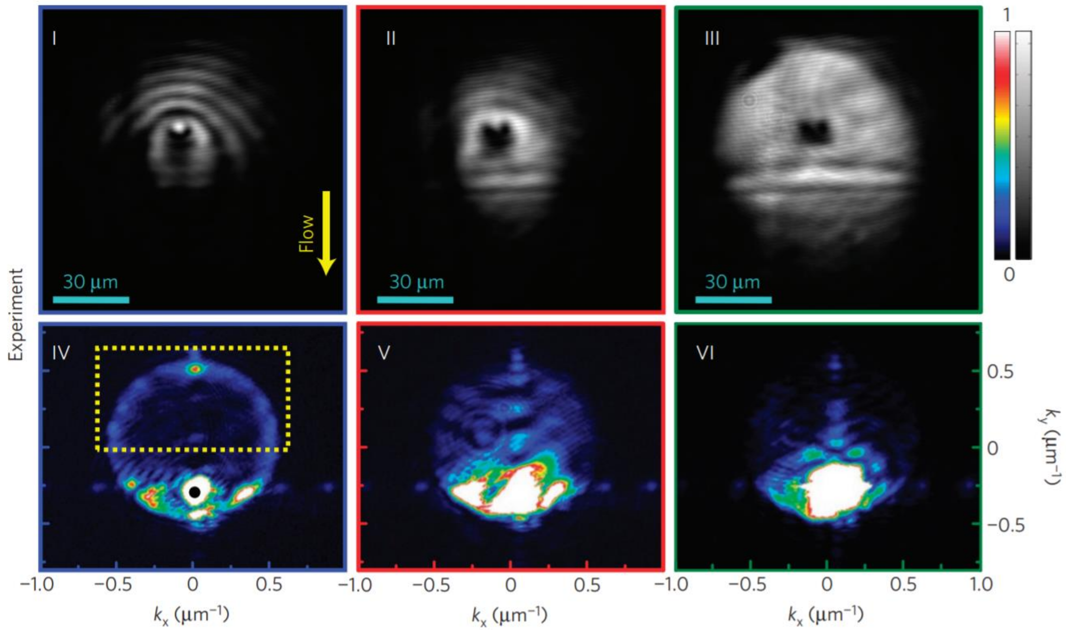


Next week's program

- Study of the stability of the solutions (Bogoliubov excitation spectrum)
- Consequences for polariton superfluidity ("quantum fluids of light")



Nature Physics **5**, 805 (2009)